Convex Optimization and Signal Processing Using Fixed-Point Characterizations of Closed Convex Sets

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Many convex optimization and signal processing problems have been solved through elegant translations into fixed point problems. The underlying principle is to operate a certain quasi-nonexpansive mapping iteratively and generate a convergent sequence to its fixed point. However, such a mapping often has infinitely many fixed points, meaning that a careful selection from the fixed point set has been desired strongly. Fortunately, considering the collection of quasi-nonexpansive mappings as a toolbox, we can select an optimal fixed point simply by the *hybrid steepest descent method* [1], provided that the cost function is smooth convex and its derivative is Lipschitz continuous. To deal with nonsmooth cost functions, we introduce an integration of the hybrid steepest descent method and the *Moreau-Yosida regularization* by highlighting its distinctive properties as a smooth approximation of a nonsmooth convex function [3]. Lastly, we introduce briefly recent powerful applications of the quasi-nonexpansive mapping to the adaptive filtering and the online machine learning problems [2,4].

References

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